EXERCISES [MAI 1.15]

TRANSFORMATION MATRICES

SOLUTIONS

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A. Paper 1 questions (SHORT)

1. (a) $x' = 2x + 5y$

$y' = x + 4y$

(b) The image of O(0,0) is O(0,0) itself.
The image of A(1,1) is A´(7,5).
The image of B(3,5) is B´(31,23).

2. (a) $M^{-1} = \frac{1}{3}\begin{pmatrix} 4 & -5 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 4/3 & -5/3 \\ -1/3 & 2/3 \end{pmatrix}$

(b) $M\begin{pmatrix} 3 \\ -2 \end{pmatrix} = \begin{pmatrix} 2 & 5 \\ 1 & 4 \end{pmatrix}\begin{pmatrix} 3 \\ -2 \end{pmatrix} = \begin{pmatrix} -4 \\ -5 \end{pmatrix}$

(c) $M^{-1}\begin{pmatrix} -4 \\ -5 \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 4 & -5 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} -4 \\ -5 \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 9 \\ -6 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$

(d) It is the line segment OA'

3. (a) The image of A(4,0) is A´(8,4).
The image of B(0,1) is B´(5,4).

(b) easy sketch!

(c) $\det M = 3$

(d) Area of OAB =2, Area of $\text{OA}'\text{B}'$ =3×2=6

4.

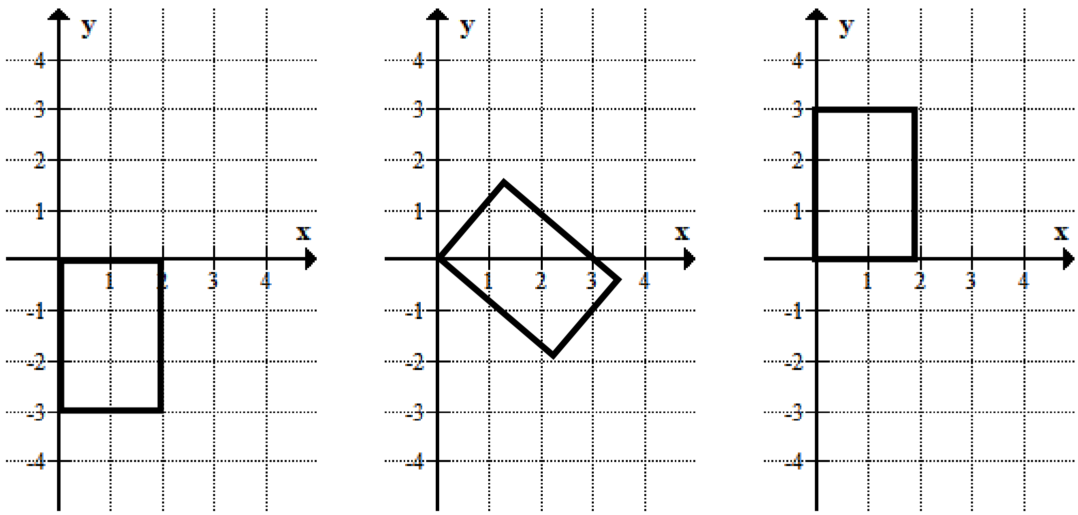
Matrix	**Description of transformation**	**New vertices**	**Area**
$\begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$	**horizontal stretch** with a scale factor of 3	O(0,0), A(0,2) B(9,2), C(9,0)	18
$\begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$	**vertical stretch** with a scale factor of 3	O(0,0), A(0,6) B(3,6), C(3,0)	18
$\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$	**enlargement** with a scale factor of 3	O(0,0), A(0,6) B(9,6), C(9,0)	54
$\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$	**horizontal stretch** with a scale factor of 2 and **vertical stretch** with a scale factor of 3	O(0,0), A(0,6) B(6,6), C(6,0)	36

5. (a) $R_{90} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$, $R_{45} = \begin{pmatrix} \sqrt{2}/2 & \sqrt{2}/2 \\ -\sqrt{2}/2 & \sqrt{2}/2 \end{pmatrix}$

(b) $T = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

(c)

(i) under R_{90} (ii) under R_{45} (iii) under T



6. (a) $\det M = 4$.

(b) Area = 4×5=20

(a) Area = 24 / 4 = 6

7. $A = \begin{pmatrix} 5 & 0 \\ 0 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $C = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$

$$CBA = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 5 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 10 & 0 \end{pmatrix}$$

8. (a) $\theta = 60°$, $A = \begin{pmatrix} -1/2 & \sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{pmatrix}$

(b) $A\begin{pmatrix} 0 \\ 2 \end{pmatrix} = \begin{pmatrix} -1/2 & \sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{pmatrix}\begin{pmatrix} 0 \\ 2 \end{pmatrix} = \begin{pmatrix} \sqrt{3} \\ 1 \end{pmatrix}$, so it is the point ($\sqrt{3}$,1)

9. (a) $A = \begin{pmatrix} 1/2 & \sqrt{3}/2 \\ \sqrt{3}/2 & -1/2 \end{pmatrix}$, $B = \begin{pmatrix} \sqrt{3}/2 & 1/2 \\ -1/2 & \sqrt{3}/2 \end{pmatrix}$

$$BA = \begin{pmatrix} \sqrt{3}/2 & 1/2 \\ -1/2 & \sqrt{3}/2 \end{pmatrix}\begin{pmatrix} 1/2 & \sqrt{3}/2 \\ \sqrt{3}/2 & -1/2 \end{pmatrix} = \begin{pmatrix} \sqrt{3}/2 & 1/2 \\ 1/2 & -\sqrt{3}/2 \end{pmatrix}$$

(b) $\theta = 15°$

10. (a) $\begin{pmatrix} 2 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix}+\begin{pmatrix} 3 \\ 2 \end{pmatrix}=\begin{pmatrix} 5 \\ 2 \end{pmatrix}$, so T maps $P(0,1)$ to $P'(5,2)$

$\begin{pmatrix} 2 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \end{pmatrix}+\begin{pmatrix} 3 \\ 2 \end{pmatrix}=\begin{pmatrix} 7 \\ 4 \end{pmatrix}$, so T maps $Q(1,2)$ to $Q'(7,4)$.

T maps the line segment PQ to the line segment $P'Q'$

(b) easy sketch!

11. (a) $A=\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}=I$, $B=\begin{pmatrix} 1 \\ 2 \end{pmatrix}$

(b) $A=\begin{pmatrix} 1 & 0 \\ 0 & 4 \end{pmatrix}$, $B=\begin{pmatrix} 3 \\ 2 \end{pmatrix}$

12. (a) $\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} -1 \\ 2 \end{pmatrix}=\begin{pmatrix} 2 \\ 0 \end{pmatrix}$, hence $-a+2b=2$ and $-c+2d=0$

$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 3 \\ -3 \end{pmatrix}=\begin{pmatrix} 0 \\ 3 \end{pmatrix}$, hence $3a-3b=0$ and $3c-3d=3$

$a=2$, $b=2$, $c=2$, $d=1$

(c) $\det A=-2$,

Area = (Area of OPQ) / 2 = 6/2 = 3.

B. Paper 2 questions (*LONG*)

13. (a) horizontal stretch with a scale factor of 2

It maps O(0,0) to O(0,0), A(4,0) to A´(8,0) and B(0,3) to B(0,3) (itself)

(b) $V=\begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$

(c) (i) $R=\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ (ii) vertices at (0,0), (3.0) and (0,-4)

(d) (i) $P=VRH=\begin{pmatrix} 0 & 2 \\ 3 & 0 \end{pmatrix}$

(ii) a horizontal translation with a scale factor of 2 (H)

followed by a clockwise rotation by 90° (R)

followed by a vertical translation with a scale factor of 3 (V)

(iii) vertices at (0,0), (3.0) and (0,-24)

(iv) $T=\begin{pmatrix} 1 & 0 \\ 0 & 6 \end{pmatrix}$

14.

Matrix	Description
$\begin{pmatrix} 5 & 0 \\ 0 & 1 \end{pmatrix}$	Horizontal stretch with a scale factor of 5
$\begin{pmatrix} 1 & 0 \\ 0 & 7 \end{pmatrix}$	Vertical stretch with a scale factor of 7
$\begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$	Enlargement with a scale factor of5.
$\begin{pmatrix} 1/2 & \sqrt{3}/2 \\ -\sqrt{3}/2 & 1/2 \end{pmatrix}$	Clockwise rotation by an angle 60°
$\begin{pmatrix} \sqrt{2}/2 & \sqrt{2}/2 \\ -\sqrt{2}/2 & \sqrt{2}/2 \end{pmatrix}$	Clockwise rotation by an angle 45°
$\begin{pmatrix} -1/2 & \sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{pmatrix}$	Reflection in line $y = \sqrt{3}x$
$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	Reflection in line $y = x$
$\begin{pmatrix} -0.6 & 0.8 \\ 0.8 & 0.6 \end{pmatrix}$	Reflection in line $y = 2x$
$\begin{pmatrix} 0.940 & 0.342 \\ -0.342 & 0.940 \end{pmatrix}$	Clockwise rotation by an angle 20°